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EFFECT OF ROTATION AND SUSPENDED PARTICLES ON THE STABILITY OF JEFFREY FLUID IN A POROUS MEDIUM

Pushap Lata Sharma & Mohini Kapalta

Department of Mathematics & Statistics, Himachal Pradesh University, Summer Hill, Shimla, India

ABSTRACT

An incompressible Jeffrey fluid heated from below in a porous medium, stability is taken into consideration, as well as how rotation and suspended particles may impact it. A normal mode analysis method has been used to create and quantitatively solve the dispersion relation. While the suspended particles are shown to destabilize stationary convection, rotation is found to assist stabilize the system. It is discovered that, depending on the situation, the medium permeability and the Jeffrey parameter can either stabilize or destabilize the system. The effects of rotation, suspended particles, Jeffrey parameter and medium permeability have all been depicted in graphs.

KEYWORDS: *Rotation, Suspended Particles, Jeffrey Fluid, Porous Medium*

Article History

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1 INTRODUCTION

Convection is considered in porous medium because it has wide range of applications in fluid flow and heat transfer as well as heat-exchanger, oil recovery, construction materials, cooling of electronics, minimising pollutant generation and medicinal treatments8. Lapwood6has researched thermal convection of fluid in porous medium and Nield and Bejan 7 have written a book on the subject.

Rotation results in the introduction of several new elements into the problem. In rotational fluid dynamics, vorticity is related to a number of findings. The thermal instability of rotating fluid layers heated from below has been discussed by Chandrasekhar2,who also demonstrated that rotation avoids the development of instability and shows asymptotic behaviour. In 13,15,18,the effect of rotation on thermal instability was investigated.

Scanlon and Segel14looked at how suspended particles affected the commencement of Bénard convection and discovered that coarse particles decreased the exponential growth rate of unstable disturbances while small particles increased it. The impact of suspended particles has been researched by 3 for more realistic boundary conditions.By heating fluid from below,11examined the impact of rotation and suspended particles in a porous media. Numerous additional researchers, including 1,4,12,16investigated convection with suspended particles.

Non-Newtonian fluid has a wide range of uses in the geophysical, chemical and biological sciences. Non-Newtonian fluids have a wide range of industrial and technical uses, which has increased interest in their research. One of the simplest non-Newtonian fluid models is the Jeffrey non-Newtonian fluid, which has a time derivative rather than a convective derivative. Jeffrey 5used below-surface heating to study the fluid layer's stability. Jeffrey parameter has a stabilising influence on stationary convection, according to recent research10on the effect of Jeffrey nanofluid in a porous media and 9on the same problem taking the effect of rotation into account. The start of Jeffrey fluid in a porous heat-generating layer has been studied in 17.

We are interested in researching the impact of rotation and suspended particles on the stability of Jeffrey fluid heated from below in porous media in the current work in light of the significance of the many applications described above.

2 Mathematical Model

Consider a Jeffrey fluid confined between two parallel horizontal planes $z = 0$ and $z = d$ saturated by layer of porous medium subjected to uniform rotation with angular velocity $\mathbf{\Omega}(0,0,\Omega)$ and gravity $\boldsymbol{g}$ force acting vertically. The physical system is heated from below such that uniform temperature $\beta\left(=\left|{}^{dT}/_{dz}\right|\right)$gradient is maintained.

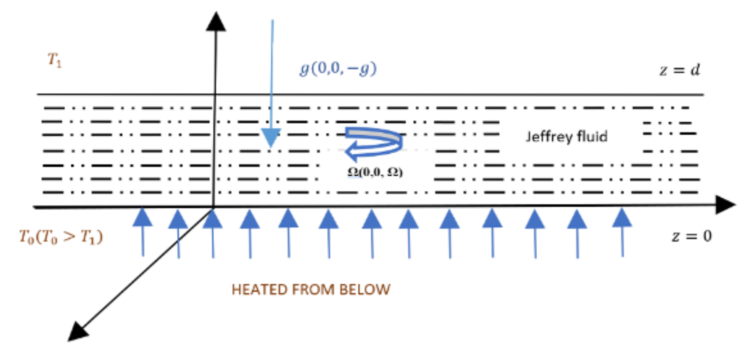

Figure 1: Physical Configuration.

The relevant equation of state, equation of continuity, equations of motion and equation of energy for Jeffrey fluid with suspended particles under rotation in porous medium under Boussinesq approximation are:

$$.\rho = \rho_0(1 - \alpha(T - T_0) \tag{1}$$

$$.\nabla . q = 0 \tag{2}$$

$$.\frac{1}{\varepsilon}\left(\frac{\partial q}{\partial t} + \frac{1}{\varepsilon}(q.\nabla)q\right) = -\frac{1}{\rho_0}\nabla p + g\left(1 + \frac{\delta\rho}{\rho_0}\right) - \frac{\mu}{k_1(1+\lambda_3)}q + \frac{2}{\varepsilon}(q \times \Omega) + \frac{KN}{\rho_0\varepsilon}(q_p - q) \tag{3}$$

where q_p is the velocity of particles,$q$is the velocity of fluid,$\varepsilon$is the porosity,$p$is the pressure,$k_1$is the medium permeability,$\lambda_3$is the Jeffrey parameter,N *is* the number of densities of the particles, $K = 6\pi\mu a$ is the constant given by Stokes drag formula, whereμ is the coefficient of viscosity and$a$ is the radius of particle.Due to the presence of suspended particles, there added an extra force term proportional to the velocity difference between particles and fluid in momentum equation (3).

The equation of energy with suspended particles is given by

$$\left(\varepsilon(\rho C)_f + (1-\varepsilon)(\rho C)_s\right)\frac{\partial T}{\partial t} + (\rho C)_f(q.\nabla)T + mNC_p\left(\varepsilon\frac{\partial}{\partial t} + (q.\nabla)\right)T = k_m\nabla^2 T$$

Using Boussinesq approximation

$$F\frac{\partial T}{\partial t} + (q.\nabla)T + \frac{mNC_p}{\rho_0 C_f}\left(\varepsilon\frac{\partial}{\partial t} + (q.\nabla)\right)T = \kappa\nabla^2 T \tag{4}$$

where, $F = \left(\varepsilon + (1-\varepsilon)\frac{(\rho C)_s}{\rho_0 C_f}\right)$ is constant,

$\kappa = \frac{k_m}{\rho_0 C_f}$ is the thermal diffusivity, k_m is the coefficient of thermal conductivity, $(\rho C)_s$ is the heat capacity of solid, $\rho_0 C_f$ is the heat capacity of fluid, $m$is the mass of the particles.

The equations of motion for particles are

$$mN\left(\frac{\partial}{\partial t} + (q_p.\nabla)\right)q_p = KN(q - q_p) \quad (5)$$

The equation of continuity for particles is

$$\frac{\partial N}{\partial t} + \nabla.(Nq_p) = 0 \quad (6)$$

There must be an additional force term in the equation of motion for the particles that is same in magnitude but opposite in sign because the force the fluid exerts on the particles is equal to and opposite from the force the particles exert on the fluid. The particles' buoyancy force is disregarded. Assumedly, the distance between the particles is considerably greater than their combined diameters. The equations describing the movements of particles (such as equation 5) have been written using these presumptions.

The initial state of the system (there is no motion) is

$$q = (0,0,0), q_p = (0,0,0), T - T_0 = -\beta z, \rho = \rho_0(1 + \alpha\beta z), N_0 = constant$$

Let $q(u, v, w), q_p(l, r, s), \delta p$ denotes the perturbations of velocity, particle velocityand pressure respectively.

Let the change in temperature distribution be $T' = T_0 - \beta z + \theta$

The change in density after perturbationθ in temperature is

$$\delta\rho = -\alpha\rho_0\theta$$

Using these perturbations in equations (2),(3) and (4) to (6) and neglecting the terms with high powers and products of perturbations, the resulting linearized perturbed equations are

$$\nabla.q = 0 \quad (7)$$

$$\frac{1}{\varepsilon}\frac{\partial q}{\partial t} = -\frac{1}{\rho_0}\nabla\delta p + \alpha g\theta - \frac{\nu}{k_1(1+\lambda_3)}q + \frac{2}{\varepsilon}(q \times \Omega) + \frac{KN}{\varepsilon\rho_0}(q_p - q) \quad (8)$$

where $\nu = \frac{\mu}{\rho}$is the kinematic viscosity.

$$(F + \varepsilon b)\frac{\partial\theta}{\partial t} = \beta(w + sb) + \kappa\nabla^2\theta \quad (9)$$

where $b = \frac{mNC_p}{\rho_0 C_f}$

$$\left(\frac{m}{K}\frac{\partial}{\partial t} + 1\right)q_p = q \quad (10$$

$$\nabla q_p = 0 \quad (11$$

Now, eliminating q_p from equations of motion for fluid (8) using equations of motion for particles (10), weget

$$.\frac{1}{\varepsilon}\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\frac{\partial q}{\partial t}=\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left[-\frac{1}{\rho_0}\nabla\delta p+\alpha g\theta-\frac{\nu}{k_1(1+\lambda_3)}q+\frac{2}{\varepsilon}(q\times\Omega)\right]-\frac{mN}{\varepsilon\rho_0}\frac{\partial q}{\partial t} \quad (12)$$

In cartesian form, equation (7),(9),(12) can be written as

$$.\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \quad (13)$$

$$.\frac{1}{\varepsilon}\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\frac{\partial u}{\partial t}=\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left[-\frac{1}{\rho_0}\frac{\partial\delta p}{\partial x}-\frac{\nu}{k_1(1+\lambda_3)}u+\frac{2}{\varepsilon}(v\Omega)\right]-\frac{mN}{\varepsilon\rho_0}\frac{\partial u}{\partial t} \quad (14)$$

$$\frac{1}{\varepsilon}\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\frac{\partial v}{\partial t}=\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left[-\frac{1}{\rho_0}\frac{\partial\delta p}{\partial y}-\frac{\nu}{k_1(1+\lambda_3)}v-\frac{2}{\varepsilon}(u\Omega)\right]-\frac{mN}{\varepsilon\rho_0}\frac{\partial v}{\partial t} \quad (15)$$

$$\frac{1}{\varepsilon}\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\frac{\partial w}{\partial t}=\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left[-\frac{1}{\rho_0}\frac{\partial\delta p}{\partial z}-\frac{\nu}{k_1(1+\lambda_3)}w+\alpha g\theta\right]-\frac{mN}{\varepsilon\rho_0}\frac{\partial w}{\partial t} \quad (16)$$

Equation (9) after eliminating s becomes

$$.\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left((F+\varepsilon b)\frac{\partial}{\partial t}-\kappa\nabla^2\right)\theta=\beta\left(\frac{m}{K}\frac{\partial}{\partial t}+1+b\right)w \quad (17)$$

Operating equation (14) by $\frac{\partial}{\partial y}$ and equation (15) by $\frac{\partial}{\partial x}$ and subtracting, weget

$$.\frac{1}{\varepsilon}\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\frac{\partial\zeta}{\partial t}=\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left[\frac{2\Omega}{\varepsilon}\frac{\partial w}{\partial z}-\frac{\nu}{k_1(1+\lambda_3)}\zeta\right]-\frac{mN}{\varepsilon\rho_0}\frac{\partial\zeta}{\partial t} \quad (18)$$

where $\zeta=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$ is the z component of vorticity.

Further, Operating equation (14) by $\frac{\partial}{\partial x}$ and equation (15) by $\frac{\partial}{\partial y}$,adding and using equation of continuity in cartesian form (i.e., (13)), weget

$$-\frac{1}{\varepsilon}\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial z}\right)=\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left[-\frac{1}{\rho_0}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\delta p+\frac{\nu}{k_1(1+\lambda_3)}\left(\frac{\partial w}{\partial z}\right)+\frac{2}{\varepsilon}(\Omega\zeta)\right]+\frac{mN}{\varepsilon\rho_0}\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial z}\right) \quad (19)$$

Now, operating equation (19) by $\frac{\partial}{\partial z}$ and using equation (16) to eliminate δp , weget

$$\frac{1}{\varepsilon}\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial t}(\nabla^2 w)=\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left[-\frac{\nu}{k_1(1+\lambda_3)}\nabla^2 w-\frac{2\Omega}{\varepsilon}\frac{\partial\zeta}{\partial z}+g\alpha\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\theta\right]-\frac{mN}{\varepsilon\rho_0}\frac{\partial}{\partial t}(\nabla^2 w) \quad (20)$$

Boundary conditions at free surfaces: At free surface tangential stresses are zero.

$$w=0,\theta=0\ at\ z=0\ and\ z=d; \quad (20)$$

$\frac{\partial^2 w}{\partial z^2}=0,\zeta=0$ on a free surface

3.Normal Mode Analysis

We attributea dependency on x,y and t of the form

$exp\left(i(k_x x+k_y y)+nt\right)$, where, k_x and k_y are the wave numbers along the $x-axis$and $y-axis$ respectively, $k^2={k_x}^2+{k_y}^2$ is the resultant wave number and n is the complex constant known as growth rate.

We consider the perturbations ofw,θ,ζ having the form'

$$[w,\theta,\zeta]=[W(z),\Theta(z),\xi(z)]exp\left(i(k_x x+k_y y)+nt\right) \quad (21)$$

Using equation (22) in equations (17), (18) and (20), weget

$$.\frac{n}{\varepsilon}\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left(\frac{d^2}{dz^2}-k^2\right)W=\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left[-\frac{\nu}{k_1(1+\lambda_3)}\left(\frac{d^2}{dz^2}-k^2\right)W-\frac{2\Omega}{\varepsilon}\frac{d\xi}{dz}-g\alpha k^2\Theta\right]-\frac{nmN}{\varepsilon\rho_0}\left(\frac{d^2}{dz^2}-k^2\right)W \quad (22)$$

$$\frac{n}{\varepsilon}\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\xi=\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left[\frac{2\Omega}{\varepsilon}\frac{dW}{dz}-\frac{\nu}{k_1(1+\lambda_3)}\xi\right]-\frac{nmN}{\varepsilon\rho_0}\xi \quad (23)$$

$$\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left((F+\varepsilon b)n-\kappa\left(\frac{d^2}{dz^2}-k^2\right)\right)\Theta=\beta\left(\frac{m}{K}\frac{\partial}{\partial t}+1+b\right)W \quad (24)$$

It is convenient to discuss equation (23)-(25) in non-dimensional variables.

$$x=x'd, y=y'd, z=z'd, a=kd, \sigma=\frac{nd^2}{\nu}, \tau=\frac{m}{K}, \tau_1=\frac{\tau\nu}{d^2}, M=\frac{mN}{\rho_0}, F_1=F+\varepsilon b, B=b+1$$

where the non-dimensional parameters are: $P_m=\frac{k_1}{d^2}$is the medium permeability, $p_r=\frac{\nu}{\kappa}$ is the Prandtl number.

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{\tau_1\sigma+1}\right)+\frac{1}{P_m(1+\lambda_3)}\right](D^2-a^2)W+\frac{2\Omega d^3}{\varepsilon\nu}D\xi+\frac{g\alpha a^2d^2}{\nu}\Theta=0 \quad (25)$$

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{\tau_1\sigma+1}\right)+\frac{1}{P_m(1+\lambda_3)}\right]\xi=\frac{2\Omega d}{\varepsilon\nu}DW \quad (26)$$

$$(D^2-a^2-E_1p_r\sigma)\Theta=-\frac{\beta d^2}{\kappa}\left(\frac{\tau_1\sigma+B}{\tau_1\sigma+1}\right)W \quad (27)$$

Eliminating Θ and ξ from equation (26) using equation (27) and (28), weget

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{\tau_1\sigma+1}\right)+\frac{1}{P_m(1+\lambda_3)}\right]^2(D^2-a^2-E_1p_r\sigma)(D^2-a^2)W+\frac{T}{\varepsilon^2}(D^2-a^2-E_1p_r\sigma)D^2W-Ra^2\left(\frac{\tau_1\sigma+B}{\tau_1\sigma+1}\right)\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{\tau_1\sigma+1}\right)+\frac{1}{P_m(1+\lambda_3)}\right]W=0 \quad (28)$$

Where $T=\frac{4\Omega^2}{\nu^2}d^4$ is the Taylor number and $R=\frac{g\alpha\beta}{\kappa\nu}d^4$ is the Rayleigh number.

Boundary conditions (21)using non-dimensional variables becomes

$$.W=0, D\xi=0 \; at \; z=0 \; and \; z=1 \quad (29)$$

and $D^2W=0, \Theta=0$ on a free surface

The proper solution of W at lowest characteristic satisfying the boundary conditions (30) must be

$$W=W_0\sin\pi z \quad (31)$$

Where W_0 is constant.

Using this solution in equation (29), weget

$$.Ra^2\left(\frac{\tau_1\sigma+B}{\tau_1\sigma+1}\right)=\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{\tau_1\sigma+1}\right)+\frac{1}{P_m(1+\lambda_3)}\right](\pi^2+a^2+E_1p_r\sigma)(\pi^2+a^2)$$
$$+\frac{\pi^2T(\pi^2+a^2+E_1p_r\sigma)}{\varepsilon^2\left(\frac{\sigma}{\varepsilon}\left(1+\frac{M}{\tau_1\sigma+1}\right)+\frac{1}{P_m(1+\lambda_3)}\right)} \quad (32)$$

Further eliminating π by letting

$$x=\frac{a^2}{\pi^2}, i\sigma_1=\frac{\sigma}{\pi^2}, R_a=\frac{R}{\pi^4}, T_a=\frac{T}{\pi^4}, P_1=\pi^2P_m, \tau_2=\pi^2\tau_1$$

Weget

$$R_a x = \left(\frac{i\tau_2\sigma_1+1}{i\tau_2\sigma_1+B}\right)\left\{\left(\frac{i\sigma_1}{\varepsilon}\left(1+\frac{M}{\tau_2\sigma_1+1}\right)+\frac{1}{P_1(1+\lambda_3)}\right)(1+x+iE_1p_r\sigma_1)(1+x)+\frac{T_a(1+x+iE_1p_r\sigma_1)}{\varepsilon^2\left(\frac{i\sigma_1}{\varepsilon}\left(1+\frac{M}{i\tau_2\sigma_1+1}\right)+\frac{1}{P_1(1+\lambda_3)}\right)}\right\} \tag{30}$$

4. Stationary Convection

Put $\sigma = 0$, for stationary convection equation (33) reduces to

$$R_a = \frac{(1+x)}{xB}\left[\frac{1+x}{P_1(1+\lambda_3)}+\frac{T_aP_1(1+\lambda_3)}{\varepsilon^2}\right] \tag{31}$$

Which expresses the modified Rayleigh number as a function of the dimensionless wave number, Taylor numberT_a, medium permeabilityP_1, porosityε, Jeffrey parameterλ_3, suspended particle parameter$B$.

To study the effect of $B, P_1, T_a, \lambda_3$and ε, we examine the behaviour of $\frac{\partial R_a}{\partial B}, \frac{\partial R_a}{\partial P_1}, \frac{\partial R_a}{\partial T_a}, \frac{\partial R_a}{\partial \lambda_3}$ and $\frac{\partial R_1}{\partial \varepsilon}$ analytically.

Equation (34) yields

$$\frac{\partial R_a}{\partial B} = -\frac{(1+x)}{xB^2}\left[\frac{1+x}{P(1+\lambda_3)}+\frac{T_aP(1+\lambda_3)}{\varepsilon^2}\right] \tag{32}$$

Which is negative so the suspended particles have destabilizing effect on the physical system.

From equation (34), we get

$$\frac{\partial R_a}{\partial P_1} = \frac{(1+x)}{xB}\left[-\frac{1+x}{{P_1}^2(1+\lambda_3)}+\frac{T_a(1+\lambda_3)}{\varepsilon^2}\right] \tag{33}$$

Thus, the medium permeability has stabilizing effect when $\frac{1+x}{{P_1}^2(1+\lambda_3)} < \frac{T_a(1+\lambda_3)}{\varepsilon^2}$ and destabilizing effect when $\frac{1+x}{{P_1}^2(1+\lambda_3)} > \frac{T_a(1+\lambda_3)}{\varepsilon^2}$.

From equation (34), we also get'

$$\frac{\partial R_a}{\partial T_a} = \frac{(1+x)}{xB}\left[\frac{P_1(1+\lambda_3)}{\varepsilon^2}\right] \tag{34}$$

This implies that rotation has a stabilizing effect on the system.

It is evident from equation (34) that

$$\frac{\partial R_a}{\partial \lambda_3} = \frac{(1+x)}{xB}\left[-\frac{1+x}{P_1(1+\lambda_3)^2}+\frac{T_aP_1}{\varepsilon^2}\right] \tag{35}$$

Thus, the Jeffrey parameter has stabilizing effect on system if $\frac{1+x}{P_1(1+\lambda_3)^2} < \frac{T_aP_1}{\varepsilon^2}$ and destabilizing effect if $\frac{1+x}{P_1(1+\lambda_3)^2} > \frac{T_aP_1}{\varepsilon^2}$.

Equation (34) yields

$$\frac{\partial R_a}{\partial \varepsilon} = -\frac{2(1+x)}{xB}\left[\frac{T_aP_1(1+\lambda_3)}{\varepsilon^3}\right] \tag{36}$$

Porosity has destabilizing effect on the physical system.

5. Principle of Exchange of Stability

Multiplying equation (26) by W^* and integrating over the range of z (i.e., from $z = 0$ to $z = 1$) and making use of equation (27) and (28), weget

$$.\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{\tau_1\sigma+1}\right)+\frac{1}{P_m(1+\lambda_3)}\right]I_1 - d^2\left[\frac{\sigma^*}{\varepsilon}\left(1+\frac{M}{\tau_1\sigma^*+1}\right)+\frac{1}{P_m(1+\lambda_3)}\right]I_2 - \frac{g\alpha a^2\kappa}{\nu\beta}\left(\frac{\tau_1\sigma^*+1}{\tau_1\sigma^*+B}\right)[I_3+\sigma^* p_r E_1 I_4] = 0 \quad (37)$$

where,

$$.I_1 = \int_0^1 (|DW|^2 + a^2|W|^2)dz$$

$$I_2 = \int_0^1 |\xi|^2 dz$$

$$I_3 = \int_0^1 (|D\Theta|^2 + a^2|\Theta|^2)dz$$

$$I_4 = \int_0^1 |\Theta|^2 dz$$

The integrals are all positive definite.

Putting$\sigma = i\sigma_i$in equation (40), and equating imaginary parts weget

$$\sigma_i\left\{\frac{1}{\varepsilon}\left(1+\frac{M}{1+(\tau_1\sigma_i)^2}\right)[I_1+d^2I_2] + \frac{g\alpha a^2\kappa}{\nu\beta}\left[\frac{\tau_1(B-1)}{1+(\tau_1\sigma_i)^2}I_3 + \frac{(\tau_1\sigma_i)^2+B}{(\tau_1\sigma_i)^2+B^2}\sigma_i p_r E_1 I_4\right]\right\} = 0 \quad (38)$$

σ_i may not always be zero. i.e.,

$$\frac{1}{\varepsilon}\left(1+\frac{M}{1+(\tau_1\sigma_i)^2}\right)[I_1+d^2I_2] + \frac{g\alpha a^2\kappa}{\nu\beta}\left[\frac{\tau_1(B-1)}{1+(\tau_1\sigma_i)^2}I_3 + \frac{(\tau_1\sigma_i)^2+B}{(\tau_1\sigma_i)^2+B^2}\sigma_i p_r E_1 I_4\right] = 0$$

Which gives the possibility that $\sigma_i \neq 0$

As a result, given the existence of rotation, suspended particles and the Jeffrey parameter, the exchange of stabilities concept may not be applicable to this issue.

6. Numerical Results

The variation of thermal Rayleigh number for stationary case with suspended particles, medium permeability, Taylor number, Jeffrey parameter and porosity for fixed wave numbers have plotted using equation (34).

Figure 2 shows the variation of $R_a$for stationary convection with suspended particles B for different values of wave number$x = 0.1, 0.5, 0.9$ and fixed values of$P_1 = 0.2, T_a = 100, \lambda_3 = 0.6$ and $\varepsilon = 0.5$. As the values of Bincreases the graph shows downward slope, thereby destabilizes the stationary convection.

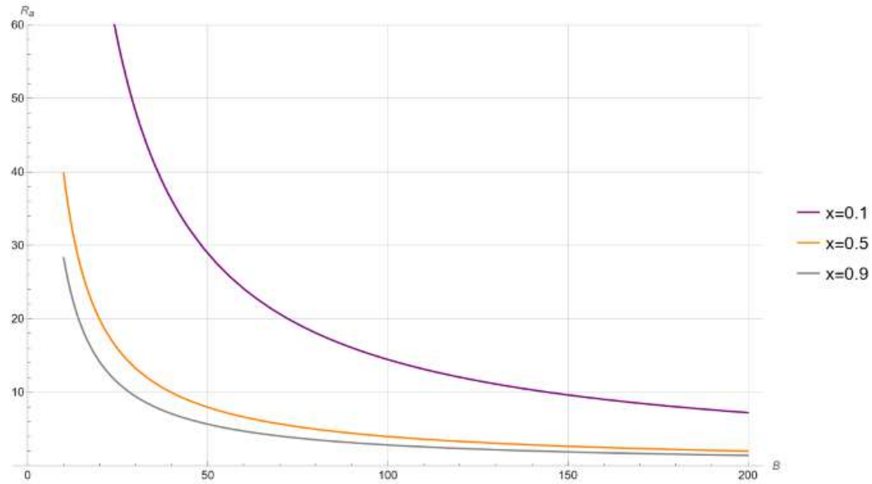

Figure 2: Variation of R_a with B for three values of wave number $x = 0.1, 0.5, 0.9$

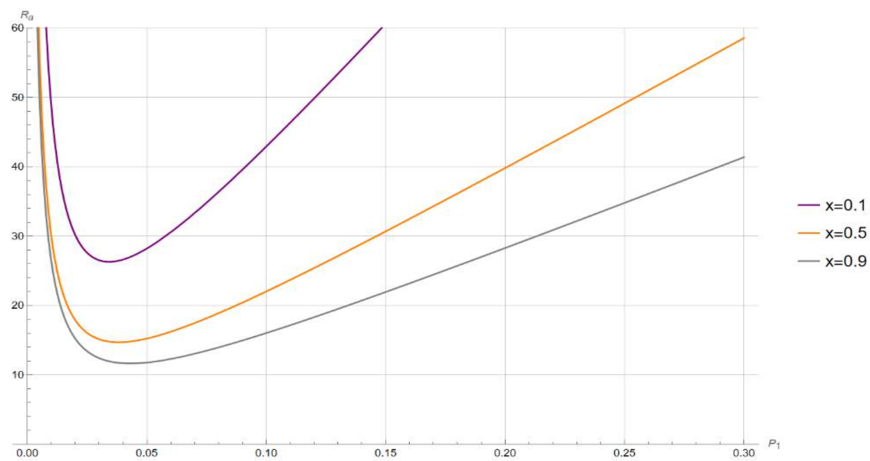

Figure 3: Variation of R_a with P_1 for three values of $x = 0.1, 0.5, 0.9$.

In figure 3, R_a is plotted against the medium permeability P_1 for different values of wave number $x = 0.1, 0.5, 0.9$ and fixed values of $B = 10, T_a = 100, \lambda_3 = 0.6, \varepsilon = 0.5$ and has both destabilizing and stabilizing effect.

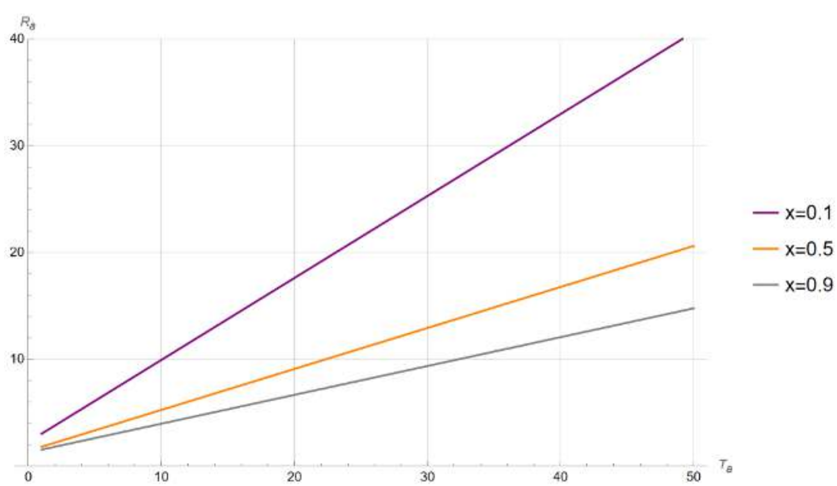

Figure 4: Variation of R_a with T_a for three values of $x = 0.1, 0.5, 0.9$.

In figure 4, R_a is plotted against the Taylor number T_a for different values of wave number $x = 0.1, 0.5, 0.9$ and fixed values of $B = 10, P_1 = 0.2, \lambda_3 = 0.6, \varepsilon = 0.5$. It is clear from the graph that effect of rotation has stabilizing effect on stationary convection.

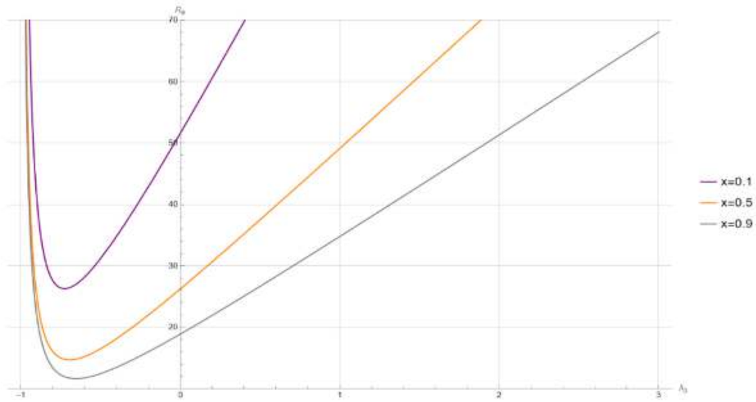

Figure 5: Variation of R_a with λ_3 for three values of $x = 0.1, 0.5, 0.9$.

In figure 5, the variation of R_a for stationary convection with Jeffrey parameter λ_3 for different values of wave number$x = 0.1, 0.5, 0.9$ and fixed values of$B = 10, P_1 = 0.2, T_a = 100$ and $\varepsilon = 0.5$. The graph shows destabilizing/stabilizing effect on the physical system.

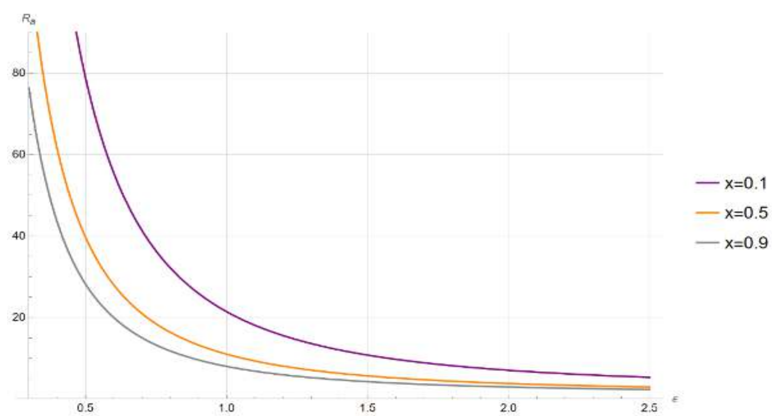

Figure 6: Variation of R_a with ε for three values of $x = 0.1, 0.5, 0.9$.

In figure 6, R_a is plotted against the porosity, $\varepsilon = 0.5$. It is clear from the graph that effect of porosity has destabilizing effect on the physical system.

7. CONCLUSION

In the current study, we investigated the linear stability theory on the start of the Jeffrey fluid by heating it from below in a porous media while taking into account the effects of rotation and suspended particles. For stationary convection, analytical and graphical solutions have been obtained and we draw the conclusion that

1. Taylor number on stationary convection show an asymptotic behaviour and has a stabilizing effect on the system.
2. The medium permeability has stabilizing effect when $\frac{1+x}{{P_1}^2(1+\lambda_3)} < \frac{T_a(1+\lambda_3)}{\varepsilon^2}$ and destabilizing effect when

$\frac{1+x}{P_1{}^2(1+\lambda_3)} > \frac{T_a(1+\lambda_3)}{\varepsilon^2}$.

3. Suspended particles $B$and medium porosityεhas destabilizing effect on the stationary convection.

4. The Jeffrey parameter has stabilizing effect on stationary convection if $\frac{1+x}{P_1(1+\lambda_3)^2} < \frac{T_a P_1}{\varepsilon^2}$ and destabilizing effect if $\frac{1+x}{P_1(1+\lambda_3)^2} > \frac{T_a P_1}{\varepsilon^2}$.

5. The principle of exchange of stabilities may not be valid in this problem due to the presence of rotation, suspended particles and Jeffrey parameter.

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